evidenced in Fig. 1(b) where the measured field intensity pattern along the horizontal median line of a mirror of aperture $2a'$ is compared with that measured along the same line but in the presence of thin rims spaced by $2a$.

The corresponding beam waveguide can be constituted by a sequence of iris surrounded by metallic annular frames, and as for the case of the dielectric frame beam waveguides [3], [4], the opaque screens can be eliminated due to the field confinement produced by the frame itself. However, this confinement is polarization sensitive.

Experimental tests have been performed on an X-band resonant section and on a prototype waveguide at 37 GHz constituted by 53 frames having aperture of 28×28 λ and spaced by 40 λ. The measured attenuation at 37 GHz is quite high due to the lack of confinement and, hence, to free propagation in the direction parallel to the electric field. Consequently, it is not possible to derive the attenuation per unit length of the guide; however, the measured total attenuation (over 53 cells) of 10 dB against 17 dB for the free-space attenuation over the same distance confirms the guidance in one plane. The tests performed on the X-band resonant section consisted of Q measurements for a variable length of the resonating section so that mirror losses and coupling losses can be taken into account. The coincidence of the Q's values in the cases of square frames and only vertical strips confirms that, due to the polarization (vertical electric field), only the vertical strips are effective. Fig. 2 shows the power losses versus number of cells for the square frames and for the vertical strips compared with the losses of an iris beam waveguide of the same size. These curves represent losses for the infinite strip case because the measured values have been corrected for the losses due to the finite size of the end mirrors in the direction of no confinement. It can be observed that this waveguide presents the same losses as the iris waveguide with the advantage of avoiding the large absorbing screen. However, the polarization sensitivity does not make this waveguide practically usable.

Other types of metallic structures can be conceived for avoiding the lack of confinement; for instance, one could use a row of parallel plates acting as waveguides which causes a suitable phase jump and gives rise to the field confinement in the vertical direction [Fig. 3(a)]. This concept can be extended to the vertical sides of the frame [Fig. 3(b)]. However, it is to be noted that in this case the structures, although metallic, are equivalent to dielectric frames because the parallel plate waveguides constituting the new metallic frame simulate a dielectric of suitable refractive index.

Measurements performed on a waveguide constituted by 53 elements like those sketched in Fig. 3(b) have given a value for the attenuation of 0.075 dB/cell or 0.21 dB/m. This is the same order as that of the dielectric frame waveguide of the same aperture. The performance of this last structure can be observed also in Fig. 4, which shows field patterns measured across the frames at two different positions along the waveguide axis in the cases of simple and multiple metallic plates, respectively. Such field patterns have been normalized with respect to the maximum amplitude after 53 elements.

In conclusion the tests have shown that the thin annular metallic frame waveguide presents the same losses as those of the iris waveguide (infinite slit), but with the advantage of a much smaller structure as the absorbing screens are no longer necessary. However, the lack of confinement in one direction due to its polarization sensitivity limits its practical use. On the contrary, practical applications can be found for the frame waveguide with multiple metallic plates which, although more complicated than the dielectric frame structure, presents the same properties.

Acknowledgment

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References

[1] P. F. Checcacci and A. M. Scheggi, "Open resonators with rimmed mirrors," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-17, pp. 125–127, Feb. 1969.
[2] P. F. Checcacci, A. Consortini, and A. M. Scheggi, "Effect of mirror rims on modes and losses of a planar Fabry–Perot resonator," *Appl. Opt.*, vol. 10, p. 1369, 1971.
[3] P. F. Checcacci and A. M. Scheggi, "Dielectric frame beam waveguide," *Proc. IEEE* (Corresp.), vol. 59, pp. 1024–1025, June 1971.
[4] P. F. Checcacci, R. Falciai, and A. M. Scheggi, "Phase step beam waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 608–613, Sept. 1972.
[5] ——, "Beam waveguides with minimized dielectric structures," *IEEE Trans. Microwave Theory Tech.* (Short Paper), vol. MTT-21, pp. 362–363, May 1973.

Bandwidth Enhancement in Dielectric-Lined Circular Waveguides

G. N. TSANDOULAS

***Abstract*—The increase in TE_{11}–TM_{01} mode bandwidth obtained by inhomogeneously loading (dielectric lining) a circular waveguide is systematically documented. Maximum bandwidth is about 31.83 percent of center frequency (up from about 26.54 percent for fully filled or empty circular waveguides). This makes circular waveguides competitive with square waveguides (bandwidth≈34.3 percent) as radiators in wide-band dual-polarization arrays. Certain interesting symmetries involving the TE_{21}–TM_{01} modal inversion are also examined.**

Introduction

Phased-array technology has been increasingly concerned with wide bandwidth performance. Radiators capable of wide bandwidth (∼60 percent of center frequency) have been described by various authors [1], [2]. These are linearly polarized rectangular-waveguide arrays in which the bandwidth limit is essentially determined by the cutoff frequencies of the first two (TE_{10}–TE_{20}) waveguide modes. For dual-polarization arrays [3] it is desirable to use radiators with at least two planes of identical symmetry such as square and circular waveguides. Based on the cutoffs of the first two (TE_{10}–TE_{11}) waveguide modes, maximum available bandwidth for square waveguides is about 34.3 percent of center frequency. For circular waveguides bandwidth is only about 26.5 percent of center, TE_{11}–TM_{01}, cutoff frequency. Thus circular waveguides compare unfavorably with square waveguides as far as maximum bandwidth is concerned. In many phased-array applications the circular radiator shape is advantageous for symmetry and for other reasons. If bandwidths in excess of about 17 percent (maximum circular-waveguide bandwidth reduced by about 10 percent to allow good matching to the exciter at the low end of the frequency band) are required, a method of increasing the available bandwidth is desirable. It has been known [4] that a dielectric lining on the inside of a circular pipe may, under proper parameter selection, increase the TE_{11}–TM_{01} bandwidth by differentially loading these two modes. A similar, if stronger, effect may also be obtained by periodically loading the guide with dielectric disks [5], [6].

In this short paper a systematic quantitative documentation of the bandwidth properties of the dielectric-lined circular waveguide is carried out. Design information for a wide variety of cases is tabulated and some new and interesting properties of the structure are presented. Meier and Wheeler's [4] good, but very limited, experimental study is used as a point of departure for this study which is carried out on the basis of the theoretical transcendental propagation equations.

Results

The geometry of the structure is shown in Fig. 1. ϵ_1 and ϵ_2 are dielectric constants relative to free space. The waveguide is described as dielectric-lined in the sense that $\epsilon_2>\epsilon_1$ with ϵ_1 not necessarily restricted to the free-space value ϵ_0. The case $\epsilon_2<\epsilon_1$ may result in complicated modal hierarchies [7] and is not of direct interest in this work. The characteristic equations for the geometry of Fig. 1 are well known [7], [8] and will not be repeated here. A computer program was written for the solutions and the following results have been obtained by means of this program.

The bandwidth figures in this short paper are given as percentages with respect to the center frequency between mode cutoffs and not with respect to the lower mode cutoff as has been usually the case in the past. This is a more realistic definition from an operational radar waveform point of view where deviation from center frequency is the meaningful quantity. This bandwidth is computed from the formula

$$\text{BW} = \text{Total percent bandwidth} = 200\left[\frac{TM_{01}{}^c - TE_{11}{}^c}{TM_{01}{}^c + TE_{11}{}^c}\right] \quad (1)$$

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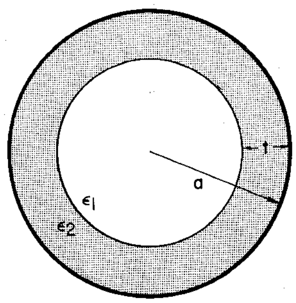

Fig. 1. Dielectric-lined circular-waveguide geometry.

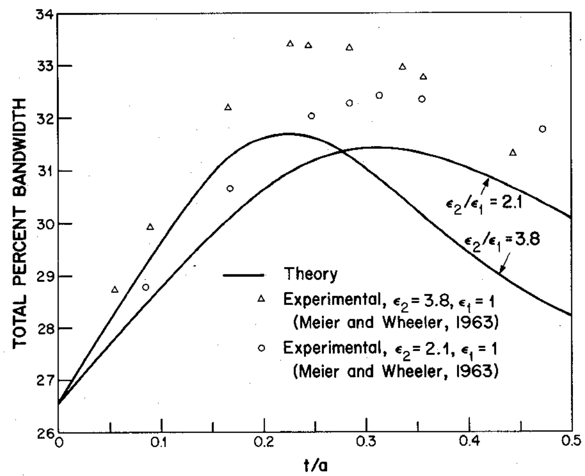

Fig. 2. Comparison of theoretical and experimental results for fused silica ($\epsilon_2 = 3.8$) and Teflon ($\epsilon_2 = 2.1$).

where TM_{01}^c and TE_{11}^c are, respectively, the cutoff frequencies of the first higher order mode (TM_{01}) and the fundamental mode (TE_{11}). The usual convention in identifying the hybrid HE_{mn} and EH_{mn} ($m \neq 0$) modes with the homogeneously filled waveguide TE_{mn} and TM_{mn} modes is followed throughout since we are dealing with cutoff conditions.

As a first step Meier and Wheeler's [4] experimental results were compared with the theoretical predictions. Fig. 2 shows this comparison. The materials used in that experiment were fused silica ($\epsilon_2 = 3.8$, $\epsilon_1 = 1$) and Teflon ($\epsilon_2 = 2.1$, $\epsilon_1 = 1$). Both abscissa and ordinate are different from Meier and Wheeler's coordinates which had to be transformed to our notation which is more directly usable. The experimentally derived bandwidth is consistently higher than the theoretically derived value. This is due to the approximate nature of the experimental method which essentially consists of extrapolating an equivalent dielectric constant (k_e) for each mode by plotting the cavity-determined straight line

$$\left(\frac{\lambda_{oc}}{\lambda_g}\right)^2 = k_e \left(\frac{\lambda_{oc}}{\lambda}\right)^2 - 1 \quad (2)$$

where λ_{oc} is the cutoff wavelength, λ the free-space wavelength, and λ_g the guide wavelength, and measuring the slope k_e. The error can be minimized by letting the y intercept, (-1), float instead of being fixed (Fig. 8 in Meier and Wheeler's paper) so as to obtain a best-fit. The greater error (about 9 percent maximum) for the higher dielectric-constant material in Fig. 2 is a natural consequence of Meier and Wheeler's fixed-intercept choice. The interesting fact from this figure is that although some disagreement exists as to the exact value of the resulting bandwidth, the variation with t/a is very similar and, more important, maximum bandwidth occurs at the same value of filling factor, t/a, for both theory and experiment.

The variation of bandwidth with filling factor follows, typically, the curves of Figs. 2 and 3. A maximum is established after which a smooth drop occurs to the empty-guide value. However, there may

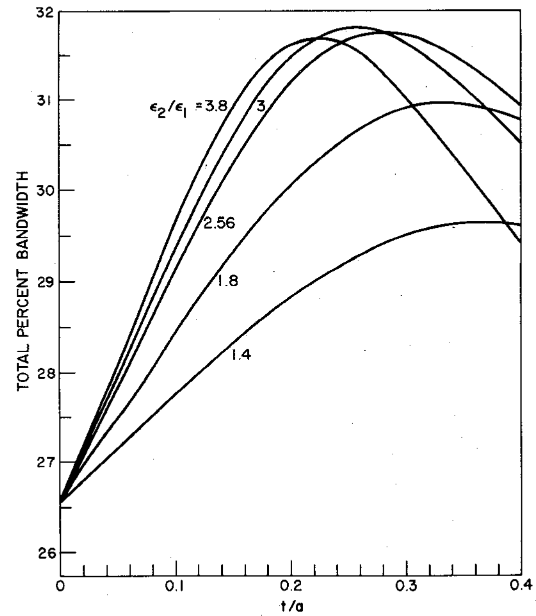

Fig. 3. Computed bandwidth curves establishing the existence of a maximum peak between $\epsilon_2/\epsilon_1 = 3.8$ and $\epsilon_2/\epsilon_1 = 2.56$.

occur one crossing of the empty-guide value somewhere to the right of the peaks shown in Fig. 3, depending on the ratio ϵ_2/ϵ_1. We will return to this point later.

Extensive calculation has shown that maximum bandwidth (31.83 percent) occurs for $\epsilon_2/\epsilon_1 \approx 3.054$ (Meier and Wheeler estimated this ratio to be about 4). The approximation sign is necessary here and in what follows since we are dealing with computer-provided solutions. The value 3.054 is interesting because, to within three decimal places, it is equal to the first zero of

$$J_2'(x) = 0. \quad (3)$$

No attempt was made to rigorously test this value by substituting it in the propagation equations. It may, however, be related to the TM_{01}–TE_{21} cutoff degeneracy to be discussed later on.

Complete design information for some commonly used dielectrics is given in Table I. Two waveguide sizes are given. One, a/λ_o, centers the bandwidth at a frequency halfway between the TE_{11}, TM_{01} cutoffs. The other, a/λ_o (+10 percent), discards the lowest 10 percent of the band as not practical (too dispersive for good match to the exciter) and centers the remaining available bandwidth.

In a phased-array application the design would ordinarily start with a specification of maximum allowable waveguide size, the restriction being imposed because of grating lobe requirements that fix the element separation. A combination of dielectric constants and filling factors may then be decided upon.

From the figures and the table it is seen that the bandwidth peaks are relatively broad so that reliable interpolation may be carried out.

The Inversion Manifold

For the homogeneously filled circular waveguide and for the results presented so far in this short paper, the modal sequence involving the first four modes is TE_{11}–TM_{01}–TE_{21}–TE_{01}. However, another sequence is possible if the ratio ϵ_2/ϵ_1 is above a certain threshold value. This sequence is TE_{11}–TE_{21}–TM_{01}–TE_{01} and is of direct interest because the TE_{21}–TM_{01} inversion reduces the bandwidth. The inversion may occur only if $\epsilon_2/\epsilon_1 > 5.135$ which is another interesting number since, to within three decimal places, it is equal to the first zero of

$$J_2(x) = 0. \quad (4)$$

It will be remembered that the first zero of the derivative of the second-order Bessel function was identified with the value of ϵ_2/ϵ_1 that maximizes bandwidth. Now we have a situation involving the

TABLE I

DESIGN DATA FOR ENHANCED-BANDWIDTH CIRCULAR WAVEGUIDES[a]

ϵ_2	ϵ_1	Max. Theoretical Bandwidth (%)	t/a	a/λ_0	a/λ_0 (+10%)
1.4 (Foam)	1.0	29.64	0.360	0.3239	0.3377
1.8 (Foam)	1.0	30.96	0.325	0.3159	0.3292
2.1 (Teflon, Foam)	1.0	31.43	0.300	0.3134	0.3266
	1.4	30.06	0.350	0.2712	0.2827
2.56 (Polystyrene)	1.0	31.76	0.275	0.3106	0.3237
	1.4	31.01	0.340	0.2648	0.2759
3.0 (Nylon)	1.0	31.83	0.260	0.3083	0.3212
	1.4	31.48	0.300	0.2642	0.2753
	1.8	30.62	0.350	0.2357	0.2457
4.0 (Boron Nitride)	1.0	31.66	0.215	0.3105	0.3235
	1.4	31.82	0.265	0.2610	0.2720
	1.8	31.56	0.300	0.2320	0.2418
	2.1	31.16	0.325	0.2164	0.2255
6.6 (Beryllia)	1.0	30.91	0.152	0.3171	0.3305
	1.4	31.46	0.200	0.2622	0.2732
	1.8	31.74	0.225	0.2313	0.2410
	2.1	31.81	0.240	0.2148	0.2238
	2.56	31.76	0.275	0.1940	0.2021
9.5 (Alumina)	1.0	30.30	0.135	0.3161	0.3295
	1.4	30.87	0.150	0.2681	0.2794
	1.8	31.28	0.175	0.2346	0.2445
	2.1	31.51	0.200	0.2148	0.2238
	2.56	31.73	0.225	0.1937	0.2019
	4.0	31.67	0.290	0.1552	0.1618
12 (Ferrite, Alumina)	1.0	29.93	0.110	0.3214	0.3350
	1.4	30.49	0.140	0.2672	0.2785
	1.8	30.91	0.160	0.2343	0.2442
	2.1	31.16	0.170	0.2170	0.2262
	2.56	31.46	0.200	0.1940	0.20
	4.0	31.83	0.260	0.1541	0.1606

[a] *Note:* λ_0 = free-space wavelength at center frequency

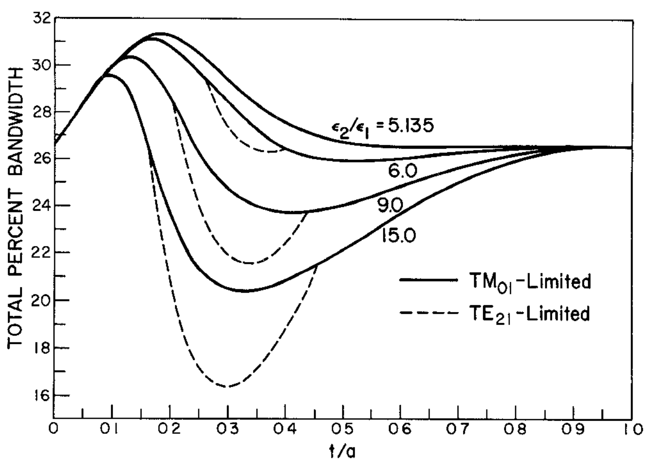

Fig. 4. Bandwidth variation with filling factor showing reduction effect by the TE_{21}–TM_{01} modal inversion.

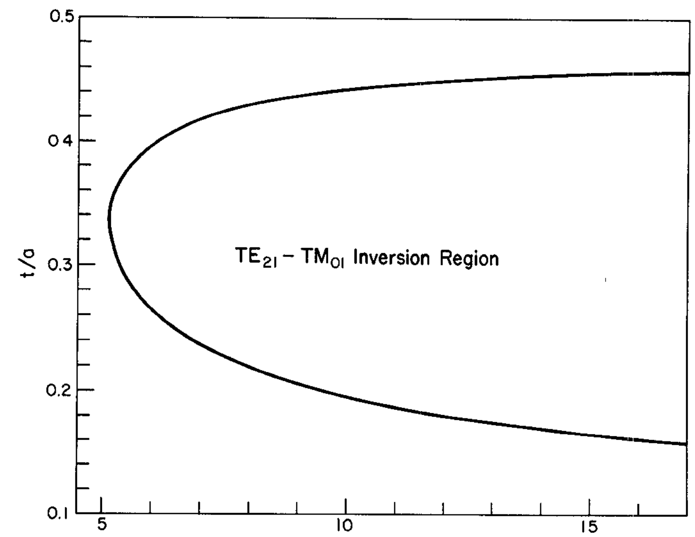

Fig. 5. TE_{21}–TM_{01} mode inversion manifold in the ϵ_2/ϵ_1, t/a plane.

first zero of $J_2(x)$ itself. Still a third curious occurrence is the fact that for $\epsilon_2/\epsilon_1 < 5.135$ the bandwidth never falls below the empty-guide value for any value of the filling factor. For $\epsilon_2/\epsilon_1 > 5.135$ the bandwidth crosses the fully filled guide value once at some intermediate t/a and then reaches a broad minimum before climbing back to the empty-guide value. This is illustrated in Fig. 4. All of these events appear too systematic to be fortuitous. However, an analytical investigation is beyond the scope of this work.

The complete inversion manifold is shown in Fig. 5 which is a plot of the locus of the TE_{21}–TM_{01} cutoff degeneracy in the t/a, ϵ_2/ϵ_1 plane. The range of parameters enclosed by the boundary lines should be avoided as it reduces bandwidth. However, as seen from Fig. 4, the bandwidth peaks always occur before the onset of inversion so that maximum bandwidth is unaffected by it. Inversion manifolds such as shown in Fig. 5 have been computed for other modes and particularly for the case $\epsilon_2 < \epsilon_1$ which leads to backward waves [9].

It appears that the two sequences TE_{11}–TM_{01}–TE_{21}–TE_{01} and TE_{11}–TE_{21}–TM_{01}–TE_{01} are the only modal sequences possible for the case $\epsilon_2 > \epsilon_1$, at least for values of this ratio up to 200.

Conclusions

The foregoing treatment demonstrates that by proper dielectric loading of circular waveguides the bandwidth advantage of homogeneously loaded square waveguides versus circular may be reduced to within 2.5 percent of center frequency ($\sim$31.8 percent for circular versus $\sim$34.3 percent for square). This effect is important in the design of wide-band phased-array radiators. Bandwidth increases above these limits may be possible for a circular waveguide loaded with three different dielectrics instead of two. However, this case appears to be too impractical for implementation. Interestingly, the full bandwidth advantage of the square waveguide cannot be restored by lining it with dielectric. In sharp contrast to the circular waveguide, the bandwidth of dielectric-lined square waveguides is always less than the fully filled guide value. A paper on this phenomenon will be published shortly.

In connection with the phased-array application it should be pointed out that the mutual coupling problem involving inhomogeneously loaded waveguides in infinite arrays has not yet been treated. In the absence of such a solution, a small array would probably have to be constructed and tested to ensure that the active element is well matched and without resonances (blind spots) within the design scan range and bandwidth.

References

[1] G. N. Tsandoulas, "Wideband limitations of waveguide arrays," *Microwave J.*, vol. 15, pp. 49–56, Sept. 1972.

[2] G. J. Laughlin, E. V. Byron, and T. C. Cheston, "Very wide-band phased-array antenna," *IEEE Trans. Antennas Propag.*, vol. AP-20, pp. 699–704, Nov. 1972.

[3] G. N. Tsandoulas and G. H. Knittel, "The design and performance of dually polarized square-waveguide arrays," *1972 Antennas Propag. Int. Symp. Dig.* (Williamsburg, Va., Dec. 11–14, 1972), pp. 157–160.

[4] P. J. Meier and H. A. Wheeler, "Dielectric-lined circular waveguide with increased usable bandwidth," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, pp. 171–175, Mar. 1964.

[5] P. J. Meier, M. A. Balfour, and H. A. Wheeler, "Circular waveguide loaded with dielectric discs for increased usable bandwidth," *IEEE Microwave Theory Tech. Int. Symp. Dig.* (Long Island, N. Y., May 19–21, 1964), pp. 33–36.

[6] P. J. Meier and S. Arnow, "Wide-band polarizer in circular waveguide loaded with dielectric discs," *IEEE Trans. Microwave Theory Tech. (1965 Symposium Issue)*, vol. MTT-13, pp 763–767, Nov. 1965.

[7] G. N. Tsandoulas and W. J. Ince, "Modal inversion in circular waveguides—Part I: Theory and phenomenology," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 386–392, Apr. 1971.

[8] P. J. B. Clarricoats, "Propagation along unbounded and bounded dielectric rods," in *Proc. Inst. Elec. Eng.*, Mon. 409E and 410E, Oct. 1960, pp. 170–186.

[9] R. A. Waldron, "Theory and potential applications of backward waves in nonperiodic inhomogeneous waveguides," *Proc. Inst. Elec. Eng.*, vol. 111, pp. 1659–1667, Oct. 1964.

Letters

Comments on "Accurate Determination of Varactor Resistance at UHF and its Relation to Parametric Amplifier Noise Temperature"

A. UHLIR, JR.

In the above paper,[1] a letter[2] written by me is cited as erroneously suggesting that the series resistance of varactor diodes varies inversely as the square of the frequency. Anyone who refers to my letter will see that the opposite is the case. I showed that such a result could be explained as a measurement artifact, the frequency-dependent transformation of losses in a distributed circuit.

My letter was written in support of an earlier hypothesis that the series resistance would not be very frequency dependent. The measurements[1] confirm that hypothesis.

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[1] K. İnal and C. Toker, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 327–333, May 1973.

[2] A. Uhlir, Jr., "Apparent frequency dependence of series resistance of varactor diodes," *Proc. IEEE* (Corresp.), vol. 51, pp. 1246–1247, Sept. 1963.